# Macedonian Society of Graph Theorists 

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## Book of Abstracts

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## Contents

Invited talk ..... 4
Peter Dankelmann : Distances in orientations of graphs ..... 4
Martin Knor : On the ratio of domination and independent dom- ination in regular graphs ..... 5
Justin Z.Schroeder : Constructing mutually orthogonal sym- metric hamiltonian double latin squares from Mullin-Nemeth starters in finite fields ..... 6
Contributed talks ..... 7
Abulfaz Abiyev : Fast Triangles Counting in a Graph ..... 7
Vesna Andova: Distance based indices on nanotubical graphs ..... 8
Viktor AndonovikJ : Community analysis in Slovenian labour network 2010-2020 ..... 9
Beti Andonovic: Optimal selection of parameters for production of Multiwall Carbon Nanotubes (MWCNTs) by electrolysis in molten salts using machine learning ..... 11
Nino Bašıć : On the Nullity of Altans and Iterated Altans ..... 13
Ismail Naci Cangul: Omega Invariant and Some Open Problems in Graph Theory ..... 15
Aleksandar T. Dimitrov : Mathematical analysis in characteriza-tion of carbon nanotubes (CNTs) produced by electrolysis inmolten salts16
Pavel Dimovski : Glimpse of Translation - Modulation Invariant Banach Spaces of generalized functions ..... 18
Mirko Petruševski : Coverability of graphs by two parity regular subgraphs ..... 19
Sijche Pechkova : Linear Algorithm For Travelling Salesmen Problem Based On Iterative Extension Of The Path With A Single Point At Each Step. ..... 20
Jelena Sedlar : Locally irregular edge colorings and the bow-tiegraph . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 21Riste ŠKrekovski : The Odd Coloring . . . . . . . . . . . . . . . . 22
Participants ..... 23
Notes ..... 27
Programme ..... 28

# Invited talk <br> <br> Distances in orientations of graphs 

 <br> <br> Distances in orientations of graphs}

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#### Abstract

An orientation of a graph $G$ is a digraph obtained from $G$ by assigning a direction to every edge. The well-known Robbins' Theorem states that every bridgeless graph has an orientation that is strong, i.e., in which between every two vertices there is a path. However, Robbins' Theorem does not give any information on the possible increase in the distances between vertices.

The oriented diameter of a bridgeless graph $G$ is defined as the minimum diameter among all strong strong orientations of $G$. The natural question if a bridgeless graph of small diameter has an orientation of small diameter was answered in the affirmative by Chvátal and Thomassen, but sharp bounds on the oriented diameter of bridgeless graphs in terms of the diameter of the graph are not known, except for graphs of diameter 2 or 3 . There are also numerous results on the relationship between the oriented diameter and other graph parameters. In this talk we survey some of these results, and present new results as well.


# On the ratio of domination and independent domination in regular graphs 

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#### Abstract

A set $S$ of vertices in a graph $G$ is a dominating set if every vertex of $G$ is in $S$ or is adjacent to a vertex in $S$. If, in addition, $S$ is also independent, then $S$ is an independent dominating set. The dominating number $\gamma(G)$ of $G$ is the minimum cardinality of an independent dominating set in $G$, while the independent domination number $i(G)$ of $G$ is the minimum cardinality of an independent dominating set in $G$.

We prove that for all integers $k \geq 3$ it holds that if $G$ is a connected $k$-regular graph, then $$
\frac{i(G)}{\gamma(G)} \leq \frac{k}{2}
$$ with equality if and only if $G$ is the complete bipartite graph $K_{k, k}$. The result was previously known only for $k \leq 6$.

This is a joint work with Riste Škrekovski and Aleksandra Tepeh and the research was partially supported by Slovak research grants APVV-170428, APVV-19-0308, VEGA 1/0206/20 and VEGA 1/0567/22 and also by Slovenian research agency ARRS, programs no. P1-0383 and projects J11692 and J1-8130.


# Constructing mutually orthogonal symmetric hamiltonian double latin squares from Mullin-Nemeth starters in finite fields 

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#### Abstract

Double latin squares-a natural extension of latin squares-were introduced in 2003 by Hilton, et al., in a lengthy paper that explored constructions of these squares with specific properties. Of particular interest are mutually orthogonal symmetric hamiltonian double latin squares of order $2 n$, briefly MOSHLS( $2 n$ ), because they cannot be constructed by simply joining together latin squares of order $n$. A proof was originally provided for the existence of $\operatorname{MOSHLS}(2 n)$ for all $n \leq 13$. In this talk, we develop a construction of MOSHLS $(2 q)$ for many prime-power values of $q$ using a special family of strong starters known as Mullin-Nemeth starters. Connections to orthogonal hamilton path decompositions of complete graphs and orthogonal 1-factorizations of complete bipartite graphs will also be explored.


# Contributed talks 

# Fast Triangles Counting in a Graph 

Abulfaz Abiyev<br>Khazar University,<br>Baku, Azerbaijan<br>vetenbaku@gmail.com

Abstract

# Distance based indices on nanotubical graphs 

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#### Abstract

Nanotubical graphs are obtained by wrapping a hexagonal grid, and then possibly closing the tube with caps. In this paper we derive asymptotics for several generalizations of Wiener index for nanotubical graphs. We introduce the measure $I^{\lambda}(G)=\sum_{u \neq v} f(u, v) \operatorname{dist}^{\lambda}(u, v)$, where $\lambda \in \mathbf{R}$ and $f(u, v)$ is a nonnegative symmetric function which is non decreasing and which depends only on $\operatorname{deg}(u)$ and $\operatorname{deg}(v)$ for a graph $G$. We show that the leading term of $I^{\lambda}(G)$ depends only on $\lambda, f(x, y)$ when $\operatorname{deg}(x)=\operatorname{deg}(y)=3$, and the circumference of the nanotube. As a consequence, we obtain the asymptotics for generalized Wiener index, Harary index, hyper Winener index, additively weigghted Harary index, generalized degree distance and modified degree distance index.


(Join work with Martin Knor and Riste Škrekovski)

# Community analysis in Slovenian labour network 2010-2020 

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#### Abstract

There is little evidence on the right approach on how to delineate the subnetworks in a labour market. The subject of research in this paper is computational influence identification of the labour force transitions between different professional occupations in the Slovenian labour network from 2010 to 2020. We use community detection algorithm to identify occupation groups and apply influence analysis on the Slovenian labour network from 2010 to 2020. This directly supports the decision-makers and employment services in identifying job opportunities for job-seekers based. The main contribution is using influence analysis to detect occupations and communities that had the most significant impact on the Slovenian labour market. The research is the first work to successfully apply community and influence analysis in the Slovenian labour network to the best of our knowledge. The paper carries several important implications, primarily highlighting the usage of existing data to increase employment levels.


(Join work with Pavle Boskoski, Bojan Evkoski, Tjaša Redek, and Biljana Mileva Boshkoska)

# Optimal selection of parameters for production of Multiwall Carbon Nanotubes (MWCNTs) by electrolysis in molten salts using machine learning 

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#### Abstract

The production and use of carbon nanotubes (CNTs) have become extremely widened within the last decade. Hence, the interest for producing non-expensive and quality CNTs is high and has motivated many research projects to date. This research considers design and development of new technology for production of MWCNTs by electrolysis in molten salts using non-stationary and stationary current regimes. The electrolysis is simple, ecological, economical, and flexible. It offers possibilities for accurate control of various parameters, such as applied voltage, current density, or temperature. We infer the underlying relationship between the parameters and the quality of the experimentally produced MWCNTs by making use of explainable tree-based Machine Learning (ML) models. We train several models in a supervised manner, where as model covariates we use the parameters of the MCWNTs, and as a target variable the quality of the produced MWCNT.


All the experimental examples in our data set are labeled by domain experts. Controlling these parameters enables high-yield production, and particularly important, obtaining MWCNTs which are up to ten times cheaper compared to other existing technologies.
(Join work with Viktor Andonovikj, Mimoza Kovaci Azemi, and Aleksandar T. Dimitrov)

# On the Nullity of Altans and Iterated Altans 

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#### Abstract

Altanisation (formation of the altan of a parent structure) originated in the chemical literature as a formal device for constructing generalised coronenes from smaller structures. The altan graph of $G, \mathfrak{a}(G, H)$, is constructed from graph $G$ by choosing an attachment set $H$ from the vertices of $G$ and attaching vertices of $H$ to alternate vertices of a new perimeter cycle of length $2|H|$. We were able to prove sharp bounds for the nullity of altan and iterated altan graphs based on a general parent graph: for any attachment set, the nullity of the altan exceeds the nullity of the parent graph by at most 2 . The case of excess nullity 2 had not been noticed before; for benzenoids it occurs first for a parent structure with merely 5 hexagons. We also exhibit an infinite family of convex benzenoids with 3 -fold dihedral symmetry (point group $D_{3 h}$ ), where nullity increases from 2 to 3 under altanisation. This family accounts for all known examples with the excess nullity of 1 where the parent graph is a singular convex benzenoid.


## References

[1 ]N. Bašić, P. W. Fowler, On the nullity of altans and iterated altans, MATCH Commun. Math. Comput. Chem. 88 (2022) 705-745.
[2 ] N. Bašić, P. W. Fowler, A curious family of convex benzenoids and their altans, Discrete Math. Lett. 9 (2022) 111-117.
(Join work with Patrick W. Fowler)

# Omega Invariant and Some Open Problems in Graph Theory 

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#### Abstract

Omega invariant is a graph parameter which is closely related to the Euler characteristic and the cyclomatic number of a graph. It has been applied to many combinatorial problems related to several graph parameters, some counting algorithms, topological indices etc. In this talk, we shall summarize some of the latest results and answer some open problems related to realizability of a given degree sequence.


# Mathematical analysis in characterization of carbon nanotubes (CNTs) produced by electrolysis in molten salts 

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#### Abstract

This research discusses full molecular characterization of carbon nanotubes (CNTs) produced by electrolysis in molten salts. Each CNT has its own mathematical representation due to its hexagonal lattice structure. Multiwall carbon nanotubes (MWCNTs) are considered and the focus is directed to determining their structural parameters: innermost and outermost diameters, chiral indices $m$ and $n$, number of walls and their unit cell parameters. Corresponding frequency parts of Raman spectra of four experimentally produced CNTs are elaborated, as well as employment of Python programming for the most accurate ( $\mathrm{m}, \mathrm{n}$ ) assignment. Determining the chirality of these samples enables calculation of other structural properties, including the graph representation of their walls' unit cells and hence further evaluation of different distance based topological indices.


(Join work with Viktor Andonovikj, Beti Andonovic, and Mimoza Kovaci Azemi)

# Glimpse of Translation - Modulation Invariant Banach Spaces of generalized functions 

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#### Abstract

We introduce and study a new class of translation-modulation invariant Banach spaces of ultradistributions. These spaces show stability under Fourier transform and tensor products; furthermore, they have a natural Banach convolution module structure over a certain associated Beurling algebra, as well as a Banach multiplication module structure over an associated Wiener-Beurling algebra. We also investigate a new class of modulation spaces, the Banach spaces of ultradistributions $\mathcal{M}^{F}$ on $\mathbb{R}^{d}$, As a classical application we investigate Wave front set with respect to translationmodulation invariant Banach spaces.


# Coverability of graphs by two parity regular subgraphs 

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#### Abstract

A graph is parity regular if all its vertex degrees are of the same parity. The talk is about coverability and decomposability of graphs into two parity regular subgraphs. We discuss complexity aspects and sufficient conditions for the existence of such covers/decompositions. (Joint work with Riste Škrekovski)


# Linear Algorithm For Travelling Salesmen Problem Based On Iterative Extension Of The Path With A Single Point At Each Step. 

Sijche Pechkova

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#### Abstract

An algorithm that gives good solutions for the travelling salesmen problem is presented. When given a large number of points every algorithm that is even $O\left(n^{2}\right)$ takes huge amount of time. Sometimes a solution that doesn't take large resources and takes weeks to run is needed. We have implemented an algorithm and tested it on a known dataset. The obtained results are satisfactory and close to optimal. This approach is useful to find good solution to the stated problem in a limited amount of time and available resources.


(joint work with Mile Jovanov and Aleksandar Pechkov)

# Locally irregular edge colorings and the bow-tie graph 

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#### Abstract

A locally irregular graph is any graph such that any two neighboring vertices have distinct degrees. A locally irregular edge coloring of a graph G is any edge coloring such that every color induces a locally irregular subgraph of G and a graph G is colorable if it admits such a coloring. Local Irregularity conjecture claims that every colorable graph requires at most three colors for a locally irregular edge coloring. We comment the validity of the conjecture on the class of unicyclic graphs and cacti where the bow-tie graph is particularly relevant.


# The Odd Coloring 

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#### Abstract

A proper vertex coloring $\varphi$ of graph $G$ is said to be odd if for each nonisolated vertex $x \in V(G)$ there exists a color $c$ such that $\varphi^{-1}(c) \cap N(x)$ is odd-sized. The minimum number of colors in any odd coloring of $G$, denoted $\chi_{o}(G)$, is the odd chromatic number. Odd colorings were recently introduced in [M. Petruševski, R. Škrekovski: Colorings with neighborhood parity condition]. In the talk we discuss various basic properties of this new graph parameter, establish several upper bounds, several charatcterizatons, and pose some questions and problems. We will also consider another new and related coloring, so called the proper conflict-free coloring.


(Join work with Mirko Petruševski and Yair Caro)

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Notes

## 6th Macedonian Workshop on Graph Theory and Applications

## PROGRAMME



|  |  | -invited talk- |
| :--- | :--- | :--- |
|  | $11: 15-11: 35$ | Sijce Pechkova <br> Linear Algorithm For Travelling Salesmen Problem Based On Iterative <br> Extension Of The Path With A Single Point At Each Step |
|  | $11: 40-12: 00$ | Vesna Andova <br> Distance based indices on nanotubical graphs |
|  | $12: 10-14: 30$ | Lunch break |
| $14: 30-14: 50$ | Pavel Dimovski <br> glimpse of Translation -- Modulation Invariant Banach Spaces of functions |  |
|  | $15: 55-15: 15$ | Aleksandar T. Dimitrov <br> Mathematical analysis in characterization of carbon nanotubes (CNTs) <br> produced by electrolysis in molten salts |
|  | $15: 45-16: 15$ | Beti Andonovic <br> Optimal selection of parameters for production of Multiwall Carbon <br> Nanotubes (MWCNTs) by electrolysis in molten salts using machine <br> learning |
|  | Coffee break |  |
|  | Riste Skrekovski <br> The Odd Coloring |  |
| $17.08 .20-16: 40$ | $16: 45-17: 30$ | Open problem session |
|  | $17: 35-17: 50$ | Wlosing ceremony |
|  | Wednesday | Conference dinner |
|  | Departure |  |
|  |  |  |

