Faculty of Electrical Engineering and Information Technologies,
Ss Ciril and Metodius University

2nd Macedonian Workshop on Graph Theory and Its Applications

14 August-18 August
Ohrid, Macedonia

2nd Macedonian Workshop on Graph Theory and Its Applications

PROGRAMME

	Monday 14.08.2017	Tuesday 15.08.2017	Wednesday 16.08.2017	Thursday 17.08.2017	Friday 18.08.2017
8:00-10:00		Breakfast	Breakfast	Breakfast	Breakfast
10:15-11:00	Arriving in Ohrid	*One open problem on Wiener index of a graph* S. Majstorović	*Borderenergetic graphs* B. Furtula	*On two conjectures on sum of the powers of signless Laplacian eigenvalues of a graph* F. Ashraf	
11:00-11:45		*Distance based indices on fullerene graphs* V. Andova	*Spanning trees and eigenvalues of graphs* E. Ghorbani	*Prime labeling of 2-regular graphs* J. Schroeder	
12:00-14:00	Lunch	Lunch		Lunch	Lunch
14:30-15:00	*Opening talk* R. Škrekovski				
15:00-15:45	*Zagreb Indices and Beyond* I.Gutman	*Mathematical Aspects of Balaban index* R.Škrekovski		*Odd 4-edge-colorability of graphs* M.Petruševski	
16:00-16:45	*Lower bound on the Szeged index using the Wiener index* M.Knor	*Predicting melting points by the Graovac-Pisanski index* P. Žigert & N. Tratnik		*Odd edge-colorability of subcubic graphs* R. Atanasov	
16:45-17:30	*Maximal Wiener index for graphs with prescribed number of blocks* K. Hriňáková	*The Stable Roommates Problem with Unranked Entries* A.Shurbevski		*Problem session*	
19:00-21:00	Diner	Diner	Diner	Diner	

Zagreb Indices and Beyond

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Let G be a graph with vertex set $\mathbf{V}(G)$ and edge set $\mathbf{E}(G)$. Let $d(v)$ denote the degree (= number of first neighbors) of the vertex $v \in \mathbf{V}(G)$. Two old (oldest) vertex–degree–based graph invariants that are of interest in mathematical chemistry and the *first Zagreb index* M_1 and the *second Zagreb index* M_2, defined as

$$M_1 = \sum_{v \in \mathbf{V}(G)} d(v)^2 = \sum_{uv \in \mathbf{E}(G)} \big[d(u) + d(v)\big]$$

and

$$M_2 = \sum_{uv \in \mathbf{E}(G)} d(u)\,d(v)\,.$$

Their mathematical properties were extensively studied in hundreds of papers.

Recently, several modifications of the Zagreb indices have been considered, of which the most interesting are the following:

Some time ago the *general first Zagreb index*

$$\sum_{v \in \mathbf{V}(G)} d(v)^\alpha = \sum_{uv \in \mathbf{E}(G)} \Big[d(u)^{\alpha-1} + d(v)^{\alpha-1}\Big]$$

was studied, where α is a real number. Special attention was paid to the cases $\alpha = -1$ (*inverse degree*) and $\alpha = 3$ (*forgotten index*).

The *hyper–Zagreb index* and the *sigma-index* are defined as:

$$\sum_{uv \in \mathbf{E}(G)} \big[d(u) + d(v)\big]^2 \qquad \text{and} \qquad \sum_{uv \in \mathbf{E}(G)} \big[d(u) - d(v)\big]^2\,.$$

The *reformulated* versions of Zagreb–type indices replace vertices with edges, and consider the degree $d(uv)$ of the edge $uv \in \mathbf{E}(G)$. Thus, the reformulated Zagreb–type index of a graph G is just the ordinary Zagreb–type index of the line graph $L(G)$.

Zagreb–type invariants, based on combination of vertex and edge degrees have also been put forward.

Let $d_2(v)$ be the second-degree of the vertex v (= number of its second neighbors). Then the *leap Zagreb indices* are

$$\sum_{v \in \mathbf{V}(G)} d_2(v)^2 \quad , \quad \sum_{uv \in \mathbf{E}(G)} d_2(u)\, d_2(v) \quad , \quad \sum_{uv \in \mathbf{E}(G)} d(u)\, d_2(v)\,.$$

To each Zagreb–type index of the form

$$Z = \sum_{uv \in \mathbf{E}(G)} F\Big[d(u), d(v)\Big]$$

a *coindex* can be associated, defined as

$$\overline{Z} = \sum_{\substack{uv \notin \mathbf{E}(G) \\ u \neq v}} F\Big[d(u), d(v)\Big] = \sum_{uv \in \mathbf{E}(\overline{G})} F\Big[d(u), d(v)\Big]\,.$$

Some properties of the above listed Zagreb–type indices will be mentioned and open problems discussed.

Lower bound on the Szeged index using the Wiener index

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(*joint work with* M. Bonamy, B. Lužar, A. Pinolu and R. Škrekovski)

Let G be a connected non-complete graph on n vertices. We prove that $Sz(G) \geq W(G) + 2n - 6$, with equality if and only if G is the complete graph K_{n-1} with one vertex attached to either 2 or n-2 vertices. Further, using our method we improve some known lower bounds on the Szeged index for special graphs, such as the bipartite graphs and the graphs of girth at least five. We also pose some related conjectures.

Maximal Wiener index for graphs with prescribed number of blocks

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(*joint work with* Stéphane Bessy, François Dross *and* Riste Škrekovski)

Wiener index, which is defined as the sum of distances between all unordered pairs of vertices in a graph, is one of the oldest and also most popular molecular descriptors. We show that among all graphs on n vertices and with given number of blocks p, the maximum Wiener index attains either a graph composed of a path with length $p-1$ with attached cycle on $n-p+1$ vertices, or a graph composed of two cycles connected by a path of length $p - 2$, or just the path P_n when $p = n - 1$.

One open problem on Wiener index
of a graph

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(*joint work with* Martin Knor and Riste Škrekovski)

The Wiener index $W(G)$ of a connected graph G is defined to be the sum of distances between all pairs of vertices in G. In 1991 L. Šoltés studied changes of the Wiener index caused by removing a single vertex. He posed the problem of finding all graphs G so that equality $W(G) = W(G - v)$ holds for all their vertices v. The cycle with 11 vertices was the unique known graph with this property. We study this problem and find graphs which Wiener index does not change when a particular vertex is removed. We show that there is a unicyclic graph G on n vertices with $W(G) = W(G - v)$ if and only if $n \geq 9$. Also, there is a unicyclic graph G with a cycle of length c for which $W(G) = W(G - v)$ if and only if $c \geq 5$. Moreover, we show that every graph G is an induced subgraph of H, such that $W(H) = W(H - v)$. This shows that the class of graphs, which Wiener index does not change when a particular vertex is removed, is rich. It gives hope that Šoltes's problem may have also some solutions distinct from C_{11}.

Distance based topological indices on nanotubical graphs

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(*joint work with* Martin Knor and Riste Škrekovski)

Nanotubical graphs are obtained by wrapping a hexagonal grid, and then possibly closing the tube with patches. In this paper we determine asymptotic for several distance based topological indices, namely the Balaban index, Sum-Balaban index, Wiener index, Gutman index, and Harary index, for open nanotubes. In all the cases, the leading term depends on the circumference of the nanotubical graph, but not on its specific type.

Mathematical Aspects of Balaban index

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(*joint work with* Martin Knor and Aleksandra Tepeh)

Balaban index is defined as

$$J(G) = \frac{m}{m - n + 2} \sum \frac{1}{\sqrt{w(u) \cdot w(v)}},$$

where the sum is taken over all edges of a connected graph G, n and m are the cardinalities of the vertex and the edge set of G, respectively, and $w(u)$ (resp. $w(v)$) denotes the sum of distances from u (resp. v) to all the other vertices of G.

In the talk, I will present some of our mathematical results on this index. First, an upper bound for the Balaban index of r-regular graphs on n vertices, and also improved bound for fullerene graphs. Next we consider graphs of order n with minimum Balaban index, and finally about the accumulation points of this index.

Predicting melting points by the Graovac-Pisanski index

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(*joint work with* Matevž Črepnjak)

The Graovac-Pisanski index, which is also called the modified Wiener index, was introduced in 1991 by A. Graovac and T. Pisanski. This variation of the classical Wiener index takes into account the symmetries of a graph. More precisely, the Graovac-Pisanski index of a graph G is defined as

$$\widehat{W}(G) = \frac{|V(G)|}{2|\mathrm{Aut}(G)|} \sum_{u \in V(G)} \sum_{\alpha \in \mathrm{Aut}(G)} d_G(u, \alpha(u)).$$

In the talk, we will present how this index can be used to predict melting points of some families of molecules, for example alkanes and polyaromatic hydrocarbons.

The Stable Roommates Problem with Unranked Entries

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The stable roommates problem (SR) and its extensions is one of the most well-studied problems in the area of matching under preference. Given a set of an even number of participants, each of which has ranked all of the others in a strict order of preference, the SR asks to determine pairs of participants such that each participant is matched in a pair, and there do not exist two participants that are not matched with each other, yet prefer each other to their assigned partners. Given a matching M, a pair of unmatched participants that prefer each other to their partners matched by M is said to be a blocking pair of M, and a matching that has no blocking pair is said to be stable. Among the various extensions of the SR the most common ones are the SR with incomplete preference lists (SRI), where "acceptable" pairs are limited by only allowing participants that have included each other in their preference lists to be matched together; and the SR with partially ordered preference lists (SRP), which introduces "indifference" in participants' preferences. In the context of indifference, more than one definition of stability are possible:

- A matching is called weakly stable if there is no pair of unmatched participants that strictly prefer each other to their matched partners,

- A matching is called super-stable if no two unmatched participants prefer each other to their matched partners, or are indifferent between each other and their respective matched partners.

There exist linear time algorithms for the SR and the SRI which give a stable matching or correctly determine that none exists. While the problem of finding a super-stable matching for an instance of the SRP (or determining that none exists) is solvable in polynomial time, the equivalent problem is

NP-hard under weak stability. Henceforth, we focus on weak stability and omit the quantifier "weak." In light of the NP-hardness results from introducing indifference in the preference lists of the SR, we restrict the notion of indifference and introduce unranked entries, such that a participant is indifferent between an unranked and any other entry in her preference list. A pair of participants u and v such that u is unranked in v's preference list will never become a blocking pair since v is indifferent between u and any other participant in v's preference list. Likewise, if u and v are matched in a matching M, then no pair including v can become a blocking pair to M. We call this problem SRU, for stable roommates problem with unranked entries. We show that the SRU is NP-complete even when

- all pairs are acceptable,

- each participant has two unranked entries in her preference list, or is an unranked entry of three other participants, and

- each participant does not have unranked entries, or is herself not unranked for any of the other participants.

On the other hand, given an SRU instance with m acceptable pairs and k unranked entries in total, we can devise an O(2km)-time algorithm to find a stable matching or determine that none exists. This algorithm is based on an enumeration procedure which for each of the O(2k) subsets of pairs including an unranked entry checks if there exists a stable matching containing that subset by applying a known O(m)-time delayed acceptance algorithm due to Gusfield and Irving for the SRI.

It is an interesting future research direction to investigate the nature of SR instances where indifference in participants' preferences are further restricted, and in particular, determine if there exists a particular class of SRU instances that are solvable in polynomial time. It seems possible that the SRU where unranked entries form a matching in the corresponding graph can be solved in polynomial time.

Borderenergetic graphs

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Energy of a graph was conceived in 1978. It originates from the theoretical chemistry and it is strongly connected with the so-called total π-electron energy of a conjugated molecule.

Let's $\lambda_1, \lambda_2, \ldots, \lambda_n$ be the eigenvalues of a graph G. Then, the energy of a graph is defined as

$$\mathcal{E}(G) = \sum_{i=1}^{n} |\lambda_i| \ .$$

Several hundreds of papers dealing with graph energy have been published so far. Numerous authors have been investigating hyperenergetic and hypoenergetic graphs. A hyperenergetic graph is one that has larger energy than a complete graph of the same order, while a hypoenergetic graph is one having energy lower than its order.

On the other hand, borederenergetic graphs are those that have the same energy as the complete graph of the same order.

$$\mathcal{E}(G) = 2\,n - 2$$

Such graphs were introduced recently. The overview of results on borderenergetic graphs and computational efforts to designate them will be outlined.

Spanning trees and eigenvalues of graphs

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Let G be a graph. We denote by $A(G)$ the adjacency matrix, by $\mathcal{L}(G)$ the line graph and by $\tau(G)$ the number of spanning trees of G. We study the interconnection between $\tau(G)$ and the multiplicities of even integer eigenvalues of $A(\mathcal{L}(G))$. Our motivation comes partly from the previous works by several authors on the connection between $\tau(G)$ and the multiplicity of the zero eigenvalue, i.e. the nullity of $A(\mathcal{L}(G))$. Doob [2] proved that the binary rank (i.e. the rank over the two-element field) of $A(\mathcal{L}(G))$ for any connected graph G of order n is $n-1$ if n is odd, and $n-2$ if n is even. This result was stated and proved in the context of Matroid Theory. Considering the rank over the real numbers, Sciriha [4] showed that the order of every tree whose line graph is singular is even and also that the nullity of the line graph of a tree is at most one. A new proof of the latter result appeared later in [3]. These results can also be deduced from Doob's work. Recently, Bapat [1] found an interesting generalization by proving that if $\tau(G)$ is odd, then $A(\mathcal{L}(G))$ has nullity at most 1. We extend these results to establish a closer connection between $\tau(G)$ and eigenvalues of $L(G)$ and $Q(G)$, the Laplacian and signless Laplacian matrices of graphs G, respectively. We prove that if G has an odd number of vertices and $\tau(G)$ is not divisible by 4, then (i) $L(G)$ has no even integer eigenvalue, (ii) $Q(G)$ has no integer eigenvalue $\lambda \equiv 2 \pmod 4$, and (iii) $Q(G)$ has at most one eigenvalue $\lambda \equiv 0 \pmod 4$ and such an eigenvalue is simple. As a consequence, we extend previous results by Gutman and Sciriha and by Bapat on the nullity of adjacency matrices of the line graphs. We also show that if $\tau(G) = 2^t s$ with s odd, then the multiplicity of any even integer eigenvalue of $Q(G)$ is at most $t+1$. Among other things, we prove that if $L(G)$ or $Q(G)$ has an even integer eigenvalue of multiplicity at least 2, then $\tau(G)$ is divisible by 4. As a very special case of this result, a conjecture by Zhou et al. [5] on the nullity of adjacency matrices of the line graphs of unicyclic graphs follows.

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On two conjectures on sum of the powers of signless Laplacian eigenvalues of a graph

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Abstract

Let G be a simple graph and $Q(G)$ be the signless Laplacian matrix of G. Let $S_\alpha(G)$ be the sum of the α-th powers of the nonzero eigenvalues of $Q(G)$. We disprove two conjectures by You and Yang on the extremal values of $S_\alpha(G)$ among bipartite graphs and among graphs with bounded connectivity.

AMS Classification: 05C50
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Prime labeling of 2-regular graphs

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Abstract

A prime labeling of a graph G on n vertices is a bijection $f : V(G) \to \{1, 2, ..., n\}$ such that $f(v)$ and $f(w)$ are relatively prime for every edge vw. It was conjectured in 1982 that a 2-regular graph, which must be a union of cycles, has a prime labeling if and only if at most one of the cycles has odd length. We will discuss how the author's recent result that every cubic bipartite graph has a prime labeling applies to 2-regular graphs, and we will close with a strategy for providing a complete resolution to the original conjecture.

Odd edge-colorability of subcubic graphs

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(*joint work with* Mirko Petrushevski and Riste Škerkovski)

An edge-coloring of a graph G is said to be odd if for each vertex v of G and each color c, the vertex v either uses the color c an odd number of times or does not use it at all. The minimum number of colors needed for an odd edge-coloring of G is the odd chromatic index $\chi'_o(G)$. In this presentation, we consider loopless subcubic graphs, and give a complete characterization in terms of the value of their odd chromatic index.