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## Book of Abstracts

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# Invited talk

## Homomorphisms of signed graphs

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### Abstract

A signed graph is a graph together with assignment of signs (+ or -) to each of its edges.

In this talk I introduce a natural extension of notion of homomorphisms of graphs to signed graphs. Few reformulation of the question and connections with homomorphisms of 2-edge-colored graphs will be presented first.

Then, after presenting a basic no-homomorphism lemma, we will try to find graphs classes under in which the necessary conditions of the no-homomorphism lemma will become also sufficient. Two examples of such results to be discussed are as follows:

Theorem 1. If maximum average degree of a simple graph  $G$  is less than  $\frac{8}{3}$ , then for any signature  $\sigma$  the signed graph  $(G, \sigma)$  maps to the signed graph on  $K_4$  with exactly one negative edge if  $(G, \sigma)$  has no loop and no digon.

Theorem 2. Any planar signed graph with no odd-cycle and no digon maps to the signed graph on  $K_{4,4}$  where a perfect matching is a set of negative edges.

The second theorem is stronger than the four color theorem and its proof uses the four-color theorem.

## Contributed talks

Andova : Distance based indices in nanotubical graphs]**Distance based indices in nanotubical graphs**

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### Abstract

Nanotubical graphs are obtained by wrapping a hexagonal grid, and then possibly closing the tube with caps. In this paper we derive asymptotics for several generalizations of Wiener index for nanotubical graphs. We introduce the measure  $I^\lambda(G) = \sum_{u \neq v} f(u, v) \text{dist}^\lambda(u, v)$ , where  $\lambda \in \mathbf{R}$  and  $f(u, v)$  is a nonnegative symmetric function which is non decreasing and which depends only on  $\deg(u)$  and  $\deg(v)$  for a graph  $G$ . We show that the leading term of  $I^\lambda(G)$  depends only on  $\lambda$ ,  $f(x, y)$  when  $\deg(x) = \deg(y) = 3$ , and the circumference of the nanotube. As a consequence, we obtain the asymptotics for generalized Wiener index, Harary index, hyper Winener index, additively weigghted Harary index, generalized degree distance and modified degree distance index.

(Join work with Martin Knor and Riste Škrekovski)





# Mathematical Approach to Strokes as an Attractor Within Communication Dynamical System

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## Abstract

Transactional Analysis as a personality theory has offered some powerful concepts to explain and improve communication between individuals. On the other hand, positive productive communication among large numbers of people, as a compound set of transactions, as an essential aspect of human survival has not been so well explained. Like the weather and other chaotic processes, group behavior is not easily understood or predicted.

It has been long suspected that in large groups of people (organizations, communities, societies), positive communication has a leading role in maintaining the duration and quality of communication.

It is the aim of this paper to relate the mathematics notion of dynamical systems to the compound system of communication. The postulate that strokes, a concept introduced by E. Berne as a way in which people recognize each other, and elaborated by Steiner as a way of exchanging information, is discussed as a concept that introduces stability into the functioning of large groups.



# Odd edge-colorability of subcubic graphs

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## Abstract

An edge-coloring of a graph  $G$  is said to be odd if for each vertex  $v$  of  $G$  and each color  $c$ , the vertex  $v$  either uses the color  $c$  an odd number of times or does not use it at all. The minimum number of colors needed for an odd edge-coloring of  $G$  is the odd chromatic index  $\chi'_o(G)$ . In this presentation, we consider loopless subcubic graphs, and give a complete characterization in terms of the value of their odd chromatic index.

(Join work with Mirko Petruševski and Riste Škerkovski.)



# **Production and Characterization of MWCNT's by Electrolysis in Molten Salt**

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## Abstract

Carbon nanotubes (CNTs) have become the greatest research challenges due to their remarkable structure and unusual properties. Therefore, a lot of growing interest is attracted for fundamental studies as well as for numerous potential applications. To date, several methods have been developed to produce high quality CNTs. Design of new electrochemical method for production of multy wall carbon nanotubes (MWCNTs) was the aim of this study, followed by a characterization protocol. The MWCNTs was produced by electrolysis in molten salt using non-stationary current regimes. Those nanostructures are produced as a result of ion intercalation during the process of electrolysis. The cation reduces at the cathode, intercalates at the electrode surface, and therefore generates a high mechanical stress that causes exfoliation. This process of intercalation and exfoliation enables electrochemical synthesis of MWCNTs to be performed. The electrochemical route offers additional possibility for accurate control of various parameters, as are applied voltage, current density, temperature and morphology of starting material. Several techniques were employed for characterization, i.e. electron

microscopy (SEM and TEM), Raman spectroscopy, thermogravimetric and differential thermal analysis (TGA and DTA). SEM and TEM images show that nanotubes are mostly of curved shape with length of  $1 \div 20 \mu m$  and diameter of  $10 \div 40 nm$ . Raman peaks indicate that the crystallinity of produced nanotubes is rather low. The obtained results suggest that formed product contains of up to 90% MWCNTs and the rest being non-reacted graphite and fullerenes. DTA curves show that combustion process of the nanotubes takes place in two stages, i.e. at  $450^\circ C$  and  $720^\circ C$ . At the lower temperature combustion of MWCNTs occurs, while at higher one fullerenes and non-reacted graphite particles burn. The main advantages of this process, compared to other commercial and lab-scale processes used for production of MWCNTs, address to the fact that it is much cheaper.

(Join work with Beti Andonović)

# Vertex geometry on nanotubes and nanotori

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## Abstract

Nanotubes are graphs obtained by wrapping a hexagonal grid, and then possibly closing the tube with patches. Similarly, nanotori are obtained by wrapping a hexagonal grid in two directions. Here we assign a coordinate to each vertex on a hexagonal grid, and respectively on a tube and on a tori. This approach might be very useful when dealing with distances on a hexagonal tube as well as on benzenoid systems and nanotori.

(Join work with Vesna Andova and Riste Škerkovski.)





# Decompositions and coverings of graphs by parity regular subgraphs

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## Abstract

Only two types of (finite) graphs are *parity regular*, i.e., have all their vertex degrees of the same parity. Such are the even graphs and the odd graphs, where a graph is said to be *even* (resp. *odd*) if all its vertex degrees are even (resp. odd). An old result of Matthews [3] established a connection between nowhere-zero flows in graphs and (edge) coverings by even subgraphs. This talk presents the current state of research on decompositions and coverings of graphs by fewest possible number of odd subgraphs. Decomposability and coverability by a combination of even and odd subgraphs are also discussed. Throughout the talk we pose various structural and algorithmic questions and problems.

(Joint work with Riste Škrekovski.)

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# On the largest interval in the image of Wiener index

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Abstract

Wiener index  $W(G)$  of a graph  $G$  is defined as the sum of distances over all pairs of vertices in  $G$ . We will consider the image  $W[\mathcal{T}_n]$  of Wiener index on the class  $\mathcal{T}_n$  of trees on  $n$  vertices. Considering the values of Wiener index on extremal trees in  $\mathcal{T}_n$  it is easily established that the size of  $W[\mathcal{T}_n]$  is at most  $\frac{1}{6}n^3 + O(n^2)$  in the case of even  $n$  and  $\frac{1}{12}n^3 + O(n^2)$  in the case of odd  $n$ . We will consider the largest interval in  $W[\mathcal{T}_n]$  and prove that its size is  $\frac{1}{6}n^3 + O(n^{5/2})$  in the case of even  $n$  and  $\frac{1}{12}n^3 + O(n^{5/2})$  in the case of odd  $n$ . Since the size of the largest interval in  $W[\mathcal{T}_n]$  is smaller than the size of  $W[\mathcal{T}_n]$ , and given the upper bound on the size of  $W[\mathcal{T}_n]$ , we obtain as the corollary the size of  $W[\mathcal{T}_n]$  too, solving thus two conjectures posed in literature.



# Implementation of Wireless Sensor Networks in Particulate Matter Reduction Using an Air Quality Monitoring System

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## Abstract

Air pollution in cities has become a significant topic of interest in recent years, mainly because of its adverse effects on human health. Air quality in urban areas can be improved in a number of ways, one of which is implementation of green walls. Plants absorb the particulate matter through their leaves and growing medium. The main goal of this paper is to present the implementation of a low-cost and energy-efficient monitoring system for air quality using a sensor network which can be easily deployed in areas of interest. Furthermore, this paper will analyze the influence of green walls on the concentration of particular matter in the air by presenting the results from an ongoing experiment. The data in this experiment was obtained through the aforementioned air quality monitoring system.



# Walks in starlike graphs

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## Abstract

It is well known that the information on numbers of walks is contained in the powers of graph's adjacency matrix. An ability to compare numbers of walks of arbitrary length between two graphs then implies inequalities between spectral radii and Estrada indices of those graphs. Comparisons of numbers of walks are usually obtained through injective embeddings of the set of walks of one graph into the set of walks of another graph, although some shortcuts are allowed from time to time. Using a variety of comparison methods, we will prove that the ordering of starlike trees by the numbers of walks coincides with the shortlex ordering of nondecreasing sequences of their branch lengths.





# On space syntax: applications of graph theory to architecture

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## Abstract

As basic mathematical structures used to visually describe relations within a set of objects, graphs had found numerous applications in many different sciences including architecture and urbanism, where these applications form a subfield named the space syntax. Graphs in space syntax are used to describe adjacency relations between different kinds of spaces in a building or a settlement, while their invariants serve to attach numerical value to various properties of spaces. In this lecture we will review methods of assigning graphs to buildings and most used invariants in their study.



# Odd colorings and coverings of graphs

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## Abstract

A (finite) graph is odd if all its vertices have odd degrees. In the talk, we present the current state of research on covers and decompositions of graphs into fewest possible number of odd subgraphs. Given a graph  $G$ , the parameters  $\chi'_o(G)$  and  $\text{cov}_o(G)$  denote, respectively, the minimum size of a decomposition and cover of  $G$  consisting of odd subgraphs. Pyber (1991) and Mátrai (2006), respectively, have shown that for every simple graph  $G$  it holds that  $\chi'_o(G) \leq 4$  and  $\text{cov}_o(G) \leq 3$ , with both bounds being sharp. Recently, we gave structural characterizations of the same inequalities for the class of loopless (multi)graphs and found infinite families of such graphs for which the bounds are achieved. Consequently, the decision problem whether  $\chi'_o(G) \leq 4$  is solvable in polynomial time. Kano et al. (2019) showed that whether  $\chi'_o(G) \leq 2$  can be decided polynomially. Atanasov et al. (2016) characterized all loopless subcubic graphs  $G$  in terms of  $\chi'_o(G)$ , showed that for this class the decision problem whether  $\chi'_o(G) \leq 2$  is solvable in linear time and hinted that deciding on  $\text{cov}_o(G) \leq 2$  might be NP-hard. The list analogues of the graph parameters  $\chi'_o(G)$  and  $\text{cov}_o(G)$ , along with other generalizations and possible new directions of related research, are also considered. Through the talk we also pose various structural and algorithmic questions and problems.

(Joint works with Risto Atanasov, Borut Lužar and Mirko Petruševski)



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# Notes











